

## CHAPTER

# 10

## Managing Bumps in the Road

Although Number Talks are a short daily routine, there is nothing routine about them. At first, they appear to be deceptively easy . . . all we have to do is put a problem on the board and ask students how they got the answer, right? But every Number Talk takes on a life of its own when students start to explain their reasoning, and there is no road map for us to follow. We need to think on our feet about what to ask and how to respond. We need to consider who is talking, who isn't, what and how to write on the board—and we need to keep all of these things in our head at once. It is no wonder that it can be hard to know what to do next and easy to feel like we are spinning our wheels and getting nowhere.

These rough spots on the road to successful Number Talks may make it tempting to abandon the whole idea, but please don't—you *can* do this! And, as you will find, any difficulties that present themselves during Number Talks offer important learning opportunities for you and your students.

The following questions and answers address thorny issues that we and many other teachers have encountered over the years. For each question, we discuss strategies that we have found to be helpful in nudging students forward. Some strategies might work in one class but not in another; some Ruth has used with success, while others Cathy has found to be helpful. We also include “temptations to resist”—common teaching strategies we have found to be counterproductive.

Good teachers can make very different decisions—just as in mathematical problem solving, you may need to fiddle around with Number Talks until you find what works best for you and your students. We hope these ideas will help you persevere in the face of the challenges that are a natural part of learning.

## Q: What if I don't understand what a student is saying?

**A:** This is an important issue for all of your students. If one student's explanation is hard for you to understand, other students probably don't understand it, either. And, since your ultimate goal is for students to listen to and respond directly to one another, it is important that all students learn to communicate clearly about mathematics so they can understand one another. This does not happen overnight. Before the adoption of the *Common Core State Standards for Mathematical Practice* (NGA/CCSSO 2010), for example, most students did not have regular opportunities to express their reasoning or present a mathematical justification. It is understandable that they would have a hard time expressing their ideas clearly during initial Number Talks.

When you find yourself in the position of not understanding what a student is saying, keep asking and rephrasing to see if you have interpreted their words correctly. You might say something like "Let me see if I really understand what you're saying. I think you. . . ."

### Strategies That Have Worked for Us

- Ask, "What I think I heard you saying was \_\_\_\_\_. *Is that what you are saying?*" Just be careful to express what you actually heard them say.
- Ask, "I want to make sure I understand what you mean. Could you please repeat that last part?"
- Ask, "Who can explain what \_\_\_\_\_ said in your own words?"

Finally, if the preceding strategies haven't helped you, you can say, "I need some more time to think about your strategy, and I'll get back to you." Then do think about it and do talk to the student. In giving yourself time to think about an idea, you are also giving the student time to think about how they might express the idea more effectively. This way, you don't have to feel nervous about not understanding what is being said, and you won't have to worry about losing other students who can't follow a cumbersome or inarticulately expressed idea or procedure.

### Temptations to Resist

*Putting words into a student's mouth or assuming you know what a student is trying to say.* This is so easy to do, especially when you are feeling the pressure of time. Keep probing the student's thinking and make it clear that it is *you* who does not understand—not that the student is doing a poor job of explaining. This will give the student confidence that his or her ideas are important to you.

## **Q: How can I get my students to move beyond the traditional algorithm when solving a problem?**

**A:** A belief that there is one best way to solve a math problem is the tradition in the United States. Even though the *Standards for Mathematical Practice* state that students should understand the meaning of quantities and not just how to compute them, the transition between knowing “what to do” and understanding “why” takes a while. This is another reason why Number Talks can be such an important learning experience—even for high school students. Coming to know that there are many ways to solve nearly any problem is ultimately liberating to students and adults alike. Understanding numbers and how operations act on those numbers is foundational to the work that high school students do in mathematics. But empowering children to reason with numbers, rather than remember what they are “supposed” to do, takes time, patience, and grit.

### **Strategies That Have Worked for Us**

- The first time the traditional US paper and pencil algorithm is offered as a strategy during a Number Talk, we explain briefly what an algorithm is.<sup>1</sup> From then on, whenever the so-called “regular” way arises as a strategy, we write *traditional algorithm* on the board. As the numbers get larger in Number Talks, students who continue to cling to these algorithms will gradually realize for themselves that other methods can be much easier and more efficient.
- We also try to find problems that are unwieldy with the traditional algorithm but easier using a different method; sometimes it takes a few tries before we find one that works. In one high school class, for example, after five unsuccessful attempts by the teacher to get students to reason in new ways, the students finally gave up the traditional algorithm when their teacher gave them this problem: “You are going to buy five milkshakes for you and your friends. Each milkshake costs \$1.99. How much did you pay for the milkshakes?”
- Sometimes we say, “Someone in \_\_\_\_\_’s class did it like this. See if you can understand what they did.” Then we choose another problem that lends itself to this new strategy so that students have a chance to try on the idea. First, ask students to share how they solved this next problem; if no one shares the new strategy, ask, “Did anyone try the method that students in the other class tried?” If no one did, then ask, “How do you think they might have used their strategy to solve this problem?”

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1. An algorithm is defined as “a precisely specified sequence of steps that will lead to a complete solution” for whichever operation you are working on (Bass 2003, 323). The problem is that the steps of the traditional algorithms have been streamlined so compactly that why they work is hidden from view when students use them. For more on computation and algorithms, see Bass (2003).

- Another strategy we use to nudge students toward reasoning is to say something like “I can see many of you used the traditional algorithm, but when you are trying to figure out something in your head, there are other strategies that are easier to understand and much easier to use. Let’s look at this problem. How else could we do this that might be easier? And how else can we think about it? . . . And how else?” Continuing to ask, “How else?” shifts the emphasis from how students have done the problem to other ways they *might* think about it. We have found that this subtle difference can engage the class in thinking differently and creatively together.

### **Temptations to Resist**

*Assuming that a student understands why a procedure works or assuming that the traditional algorithm is the best way to do a problem.* Expecting students to explain why a standard algorithm works generally isn’t productive because most students who learned mathematics as procedures have not been asked to make sense of them in the first place. This is not to say that understanding the logic and mathematical principles underlying these algorithms is not important. Rather, we have found that once children have brought meaning to operations through their own reasoning, the procedures in these traditional algorithms become more readily transparent.

When a student uses the traditional algorithm, it is okay to ask a few “Why?” questions, but, as discussed, you won’t want to dwell on the “why” if students revert back to “what.” Sometimes we remind students about the goal of Number Talks, saying, for example, “With Number Talks, remember that we are trying to use strategies that we can make sense of and that make it easy to reason with numbers. Can you think of a way to solve this problem that is easier for you?”

**Q: *Already, in the third week of school, there are a few students who want to share almost every day, while many other students have never shared at all. How can I get more students involved?***

**A:** This happens in every classroom. Some students naturally enjoy sharing their ideas, while others do not. Why? First, talking to the whole class takes experience, which many of our students have not had, and confidence, which many of our students lack. Students may not have an answer or may be afraid their answer is wrong. They may feel a lack of confidence with their English. They may be afraid they won’t be able to explain how they got their answer. They may be generally shy. Some students might have had very little experience explaining their ideas at home, while others talk constantly with their parents.

There is much talk about the importance of a safe classroom culture, and we of course

agree. Students need to have trust—in the teacher and in the other students in the class—that even their fledgling and incomplete ideas will be respected. But while a safe culture is essential, it is not sufficient to ensure all students' participation.

The dilemma is that explaining and justifying are essential for *all* students. Knowing they can be called on to speak at any moment, though, changes the nature of what people are able to think about. Many students find that the potential to be called on at any moment can easily shift their attention from thinking deeply about the topic under consideration to whether and when they will be chosen to speak, with varying levels of anxiety, depending on how prepared they feel.

We work diligently behind the scenes to encourage reticent students to begin to share their ideas, and even so we are rarely 100 percent successful. But we always work from the premise that all students will have control of whether or not to publicly contribute to the classroom discourse.

### Strategies That Have Worked for Us

- We often say, “I would like to hear from someone who hasn’t had a chance to share.” Then we *wait* . . . and wait some more. You may find that silently counting to ten or twenty helps with what might seem like an interminable amount of time. If no one volunteers after a long wait time, try a different approach, but come back to this strategy in subsequent Number Talks.
- Sometimes bringing a small group of students together in a private Number Talk can be effective in helping them develop confidence in their ability to explain their thinking. Inviting students to share their strategies in a small group can be a safer way for students to learn to share their thinking with others, and they often come to realize that their ideas are valuable. We know this is easier in a self-contained elementary classroom than in a middle school or high school math classroom where students come and go every hour. It is hard to keep track of who is talking and who isn’t when you have 150 students. So be patient with yourself, but also be persistent about gradually having a quick individual or small-group Number Talk with students you haven’t heard from. Sometimes that’s all it takes to get a student to more actively participate in whole-class discussions.
- In classes of painfully reticent older students, some teachers have had success having students share their strategies with a neighbor before the whole class is invited to share. But a caution here is that setting up a situation where all students are expected to share can be frightening for students who are tentative in their thinking, even if they only have to share with one or two other people. Another caution is that if students share their answers before being asked to give answers to the whole class, everyone can

be deprived of the rich possibilities provided by different answers in a Number Talk.

- Sometimes we do a quick formative assessment. First we pose a problem and ask students to mentally solve it two different ways. Then we ask students to record their strategies on both sides of a 3-by-5-inch card. Once everyone is finished, students share within their small groups. Often students who don't speak to the whole class are willing to risk sharing their ideas with a few other students. Smaller groups also offer an easier setting for students to ask questions about another method. When the small-group discussions begin to wind down, we have students put their names on the cards, and we collect them. This short process can also serve as an invaluable formative assessment to help you choose what direction to take in the next Number Talk.

Our bottom line is that we want the learning environment to be safe for all students. Ruth shares,

I tell my students on the first day of class that I won't put them on the spot but that I will give them lots of opportunities to share their thinking when they choose to. I try hard not to violate this trust. I want the learning environment to be safe for all students. I do talk with kids about how important it is for them to talk about and explain their thinking. With quieter students, I sometimes ask them, one-on-one, to share with me how they thought about a problem. Once they have had a chance to rehearse their thinking with me, I ask them to think about whether they might be willing to share their idea with the class. If they don't choose to, I often ask if they are willing to let me share their idea. Once students have had their own way of thinking recognized and valued, they may become more confident in sharing their ideas.

Some students, despite our best efforts, may never want to share with the whole class, and we have to be okay with that as long as we know that they are learning. That said, Number Talks are one of the safest ways we know to encourage participation.

## Temptations to Resist

We've observed some practices over the years that inhibit classroom talk. We have found that avoiding these practices, over time, results in more equitable and thoughtful classroom conversations.

- Allowing children to indicate, or "vote," verbally or nonverbally (for example, with

hand signals) when they agree with an answer can be counterproductive. Even though the intention may be to support others, we think this practice places too much emphasis on the answer rather than the problem-solving process, and ultimately can lead to a less safe environment for testing out new ideas.

- While efficiency is foundational for numerical fluency, having students choose the “best” or even “most efficient” strategy can diminish students’ willingness to explore and understand new ways of reasoning with numbers. Once students have developed confidence and flexibility with their reasoning, attending to efficiency can be a valuable pursuit.
- Remarks intended to be encouraging, or in any way indicating a preference for one strategy over another, such as “Great!” and “Yes!” put the teacher back in the spotlight. A central goal of Number Talks is to build students’ sense of agency and to help us, as teachers, stand aside for students’ thinking to take center stage. And of course those students who don’t get this kind of response can feel that their contributions are not valued or valuable.
- “Equity sticks” are tempting but counterproductive. See “Guiding Principle 6” in Chapter 3.
- Number Talks that go on too long cause many students to become restless and disengaged. With some exceptions (see Chapter 9), Number Talks should not be longer than about fifteen minutes. This means that sometimes you will need to cut a Number Talk short without hearing from students who want to contribute, but if you are doing Number Talks consistently and frequently, there will always be another chance.
- If Number Talks are sporadic, many students lose the capacity to build on ideas they have had themselves or heard from other students. New ideas are forgotten with gaps in time. Quieter students don’t have opportunities to consider new strategies and think about how they work—the kinds of experiences that can build their confidence to try out ideas publicly.
- If Number Talks are to have the intended impact, then purposeful planning must go into selecting problems; otherwise, they can become just another activity. Jumping from one kind of problem to another, and never letting students test out new ideas that they’ve seen or heard, is unlikely to result in students who can deal flexibly and confidently with numbers. We have written Chapters 4 through 8 to help you with choosing problems that build upon one another.

### **Q: What if I get confused or make a mathematical mistake?**

**A:** This can happen to anyone when mathematical ideas are being explored! For many of us, learning arithmetic was a rote process that rarely offered the opportunity to understand operations and numerical relationships deeply, so it is logical that there will be many soft spots in our own understanding. Number Talks offer us, as well as our students, the opportunity to strengthen our understanding. But, as teachers who have learned that our job is to explain and to be the source of knowledge for students, we may initially panic when we make a mistake or get confused. It isn't comfortable! But cognitive dissonance is necessary for learning—not just for our students. Mistakes—even our own—are truly sites for learning (Hiebert et al. 1997). So we must learn to take a deep breath and embrace the opportunity to model for our students a spirit of wondering, of curiosity, and of being willing to embrace and examine our mistakes in a search for meaning. We have found that it is beneficial for our students to see their teacher's comfort with being confused or wrong. We have often invited our students to help us figure out what was wrong with our thinking or our recording of an idea and have found that students of all ages willingly rise to the occasion.

### Strategies That Have Worked for Us

- “Yikes. I think there is something wrong here. Can anyone help me figure this out?”
- “Uh-oh. I just confused myself. Hmm—where am I? Can anyone help me think about this?”
- “I think I have just made a mistake—my synapses are firing!”
- “I’m going to think about this overnight, and I hope you will help me think about it. We’ll talk about it tomorrow and try to figure it out.”

### Temptations to Resist

Being embarrassed about our mistakes or trying to cover them up or make excuses for them—or saying we made a mistake “on purpose”—sends the wrong message. We are models for our students, and if we want our students to learn from their mistakes, we have to be willing to do the same. The key is being open and honest with students.

**Q:** *I know we are supposed to use mistakes as sites for learning, but what should we do when a student's answer—or method—is wrong?*

**A:** Sometimes as many as four—or more—different answers emerge from a Number Talk. Some arise from small computation errors, while others indicate misconceptions about how a property or operation works. These latter errors are the ones that offer the greatest opportunity for moving students' understanding of mathematics forward. Our goal here is to get

our students to the point where they are genuinely curious, rather than embarrassed, about their mistakes.

### Strategies That Have Worked for Us

- We establish a class norm that any answer, right or wrong, must be justified. A student explaining a strategy should begin with identifying the answer he or she is defending. “Which answer are you defending?” is a good prompt for this and communicates that the logic of the mathematics will determine whether a strategy is sound.
- Usually it becomes apparent early on which answer is “right.” There are several possible ways to approach the other answers if you decide it would be valuable to discuss them. You can ask, “Is the person who answered \_\_\_\_\_ willing to tell us how you thought about this?” If no one volunteers, you can just let it go, or you might want to ask, “How might someone arrive at this answer?” If there is a lingering question about which answer is right, you might want to say about the strategy or strategies, “Let’s try this with smaller numbers that we know the answer to and see if it works.”
- Another approach is to turn the mistake into a class investigation (see Chapter 9).

### Temptations to Resist

- Resist acknowledging, verbally or nonverbally, that an answer is right or wrong, especially before your students have had a chance to examine and defend the various answers. This may seem, for some of you, like educational malpractice. Isn’t it a teacher’s job to help students know where they have gone wrong? The reality, though, is that as students listen to different ways of solving the problems, they will notice their errors on their own, and they will work to correct their thinking. It is important to give them a chance to do this. Again, once a safe environment has been established, it is okay to ask, “Does anyone who now knows their answer is wrong want to share what you did?” If you don’t get volunteers, though, it is important not to put individual students on the spot.

### **Q: What do I do if I don’t know how to record a student’s thinking?**

**A:** Recording is trickier than it looks, and it isn’t an exact science. The main thing is to write just enough so that the class can clearly “see” how a student’s strategy works. Write too little and we probably haven’t pressed the student enough for why their method makes sense; write too much, and the student’s thinking gets muddled and/or hard for others to follow.

There are many effective ways to record, but because the recording issues are a little dif-

ferent for each operation, we have included examples of ways to record students' thinking in each operations chapter (see Chapters 4 through 8). We hope you will find these useful as you experiment with what works for you.

### Strategies That Have Worked for Us

We have found that it helps to listen to a student for a while before we start writing, in order to get the gist of the strategy or where the mathematics is going. Sometimes we erase what we have written as a student talks and start all over again. If we get really stuck on how to record the strategy, we say so, and occasionally even invite the student to come to the document camera or board to show the strategy.

### Q: *How do I get buy-in from my high school students?*

**A:** Number Talks help students of all ages become responsible for their own reasoning. But high school students have had a long time to practice what they believe to be their responsibilities in math class; typically, these include listening closely to the teacher so they will know how to follow steps for “the” way to solve a problem. But Number Talks change the rules for what is expected of them. When a teacher asks, “Why does that make sense?” students can become bewildered, mixing up what *makes sense* with what they are supposed to *do*. Sometimes they become frustrated that, all of a sudden, knowing what to do isn't enough.

Unfortunately, there is no magic wand to help students realize that making sense of what they do empowers them to be able to use mathematics in many situations. So, as you help students along this path, one important thing to remember is that success is a great motivator, and nothing feels quite as good as really understanding something. That said, we have found a couple of things that can help hook students' interest and shift their thinking.

### Strategies That Have Worked for Us

- First, we have found that starting with “dot talks” (see the classroom lesson in Chapter 2) is very valuable for all students, from elementary through high school. Dot talks do not carry the baggage of an arithmetic operation. It is fun to visualize the dots, and the students get experience with the important mathematical practice of clearly explaining their thinking in a low-risk setting. This is also an important initial step in helping students understand that people see—and think—in different ways. More importantly, dot talks level the playing field, so to speak. People see differently, and everyone can talk about what they see. Several dot talks, not just one, lay a firmer foundation for using mathematical practices in a no-risk environment.

- Second, we have seen teachers catch older students' attention with contexts they relate to, as with the milkshake math problem mentioned earlier. We also have seen teachers occasionally embed Number Talks in other contexts; one teacher, for example, did a Number Talk involving subtracting from 90 degrees or 180 degrees when her class was studying complementary and supplementary angles. On the other hand, if you want to spend just fifteen minutes in Number Talks, you will often want to present computation problems without a context (so-called naked-number problems) so that the entire focus is on how numbers and operations work and why. When you do use a context for Number Talks, it should not be so complex as to draw students' attention away from working with numbers.
- A final word about buy-in: patience. Don't give up. Keep the talks short, less than fifteen minutes, so that they hold students' attention. And be encouraging; success is a great motivator.

### Temptations to Resist

*Giving in and giving up.* Number Talks provide a foundation with numerical and algebraic reasoning, and there is too much to be gained to give up on them, on your kids, or on yourself.

Number Talks help to establish a culture where students are expected to make sense of mathematics in their own ways, learn to defend their ideas mathematically, and learn to listen to and build on the thinking of their peers. These characteristics are the essence of the *mathematical practices*, the characteristics and dispositions that will prepare students for future success with mathematics, for college, and for their careers.

**Q: My Number Talks are going pretty well, but they still seem to be two-way communications between me and a student (T-S-T-S-T-S). How can I get students to listen and talk to one another?**

**A:** This question gets to the heart of Number Talks. Our goal is to help students learn to make mathematically convincing arguments in support of their ideas and listen to and build on one another's ideas. This often means that we need to work purposefully to change the pattern of our interactions with students and theirs with one another.

### Strategies That Have Worked for Us

- Orient students to others' thinking by asking questions like "What questions do you have for Miki?" or "Do you think Ellie's method will work every time? Talk to her about what you think." When hands are raised, don't call on a student yourself, but rather, say, "\_\_\_\_\_, there are some kids who want to talk to you." Then let the one sharing

the strategy call on someone.

- You can help your students learn to ask, “Does anybody have a question?” or “Does that make sense to you?” after they have shared a way of thinking. If a student is willing to share something he or she is not sure about, this is something to be celebrated.
- You can respond by saying, “\_\_\_\_\_ is not sure her idea makes sense, so she’s asking for our help, so pay careful attention here.” We want to build a community of learners where students know that we are all in this together, trying to learn new ways of working efficiently with numbers.

### Temptations to Resist

*Being too quick to respond.* Students are used to waiting for us to respond to ideas, and we’re used to responding. It’s important to wait after a student shares to give other students a chance to think about questions they might have. This wait time also allows the student sharing to think about his or her response and perhaps refine it.

**Q: One of the hardest things for me is knowing what questions to ask! How can I get past, “Who thought about it a different way?”**

**A:** This is another important question. Number Talks are not really about how many ways problems are solved; they are about understanding how students make sense of the problems we pose. The questions we ask can help students come to value this as they learn to ask questions of one another.

### Strategies That Have Worked for Us

- After a student has shared a strategy, we often ask if anyone solved it the same way, noting students’ responses. As mentioned, we also ask the students, “What questions do you have for \_\_\_\_\_?” We first listen to students’ questions before asking our own. When students don’t ask any questions of one another, we often say, “Well, I have a question,” then we ask questions like “Why did you do \_\_\_\_\_?” or “Can you explain what you were thinking here?” Even though we might fully understand a strategy that has been shared, we often think about important ideas in the strategy that might be confusing to some students; for example, we would ask a purposeful question such as “Why did you add four here?” when a student has subtracted a multiple of ten and then added some back.
- We also sometimes ask students to consider what is similar and/or different about two strategies that have been shared by their classmates.

Kathy Richardson, who developed the practice of Number Talks with Ruth and first named them “Number Talks,” says,

The power in the Number Talks comes from inspiring each child to think and make sense of the mathematics they are presented. They are never trying to figure out what the teacher wants. Rather, they are totally engaged in their own sense-making process. . . . A Number Talk is an opportunity for children to learn that they can figure things out for themselves in the way that makes sense to them. This is the true meaning of “lifelong learner.” (www.mathperspectives.com)

We know that once you are on your way, you are likely to find that often during Number Talks your students are teaching you and their classmates more than you are teaching them, which does wonders for their sense of themselves as mathematical thinkers.

Probably the most important thing you can do for yourself is to find a colleague to collaborate and talk with as you embark on this journey. While there will likely be bumps in the road, over time, the benefits to you, your students, and your classroom as a community will far outweigh the challenges.