

Building Fact Fluency

A TOOLKIT FOR ADDITION & SUBTRACTION

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CORRELATION TO

**Idaho Content Standards
for Mathematics**

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Idaho State Standards for Mathematical Practice

– STANDARDS FOR MATHEMATICAL PRACTICE, GRADES K–12 –

<p>Idaho Mathematical Practice 1: Make sense of problems and persevere in solving them.</p>	<p><i>Building Fact Fluency:</i> Embedded throughout the toolkit, especially in these routines:</p> <ul style="list-style-type: none"> · Contextualized Practice Problems · Anchor Problems · 3-Act Math Tasks Journaling and Reflection
<p>Idaho Mathematical Practice 2: Reason abstractly and quantitatively.</p>	<p><i>Building Fact Fluency:</i> Embedded throughout the toolkit, especially in these routines:</p> <ul style="list-style-type: none"> · Image Talks · Tool Talks · Number Talks · Contextualized Practice Problems · Anchor Problems · 3-Act Math Tasks · Games · Journaling and Reflection
<p>Idaho Mathematical Practice 3: Construct viable arguments and critique the reasoning of others.</p>	<p><i>Building Fact Fluency:</i> Embedded throughout the toolkit, especially in problem-based lesson discussions, student representations, and the following routines:</p> <ul style="list-style-type: none"> · Image Talks · Tool Talks · Number Talks · Contextualized Practice Problems · Anchor Problems · 3-Act Math Tasks
<p>Idaho Mathematical Practice 4: Model with mathematics.</p>	<p><i>Building Fact Fluency:</i> Embedded throughout the toolkit, especially in these problem-based routines:</p> <ul style="list-style-type: none"> · Contextualized Practice Problems · Anchor Problems · 3-Act Math Tasks
<p>Idaho Mathematical Practice 5: Use appropriate tools strategically.</p>	<p><i>Building Fact Fluency:</i> Embedded throughout the toolkit, especially in these routines:</p> <ul style="list-style-type: none"> · Tool Talks · Games · Anchor Problems · Contextualized Practice Problems

<p>Idaho Mathematical Practice 6: Attend to precision.</p>	<p><i>Building Fact Fluency:</i> Embedded throughout the toolkit, especially in these routines:</p> <ul style="list-style-type: none"> · Image Talks · Tool Talks · Number Talks · Contextualized Practice Problems · Anchor Problems · 3-Act Math Tasks · Games
<p>Idaho Mathematical Practice 7: Look for and make use of structure.</p>	<p><i>Building Fact Fluency:</i> Embedded throughout the toolkit, especially in these routines:</p> <ul style="list-style-type: none"> · Image Talks · Tool Talks · Number Talks · Games · Journaling and Reflection
<p>Idaho Mathematical Practice 8: Look for and express regularity in repeated reasoning.</p>	<p><i>Building Fact Fluency:</i> Embedded throughout the toolkit, especially in these routines:</p> <ul style="list-style-type: none"> · Image Talks · Tool Talks · Number Talks · Anchor Problems · Games · Journaling and Reflection

Idaho Content Standards for Mathematics – KINDERGARTEN –

Counting and Cardinality – K.CC	
<p>K.CC.A Know number names and the count sequence.</p> <ol style="list-style-type: none"> 1. Count to 100 by ones and by tens. 2. Starting at a given number count forward within 100 and backward within 20. 3. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects). 	<p>Students have opportunities to develop understanding of place value and numbers within 100 in many tasks in the <i>Building Fact Fluency</i> toolkit, especially in the Contextualized Practice Problems, which provide number choices within 5, within 10, within 20, and with multidigit numbers to 100. Recitation is not an explicit component of the kit, but students will make connections between counting and addition and subtraction and explore place value by composing and decomposing tens within meaningful contexts.</p>
<p>K.CC.B Count to tell the number of objects.</p> <ol style="list-style-type: none"> 4. Understand the relationship between numbers and quantities; connect counting to cardinality. 5. Given a group of up to 20 objects, count the number of objects in that group and state the number of objects in a rearrangement of that group without recounting. Given a verbal or written number from 0–20, count out that many objects. 	<p>The 94 Image and Tool Talks each offer students opportunities to count up to 20 objects in different configurations.</p> <p>In the 31 Number Talks, students work without objects and pictures. In all the problem-based lessons (Anchor Problems, Contextualized Practice Problems, 3-Act Tasks), students read, write, and represent 0–20 and beyond objects as they solve contextualized problems.</p> <p>Games offer opportunities for students to practice reading numbers and connecting representations to numbers (e.g., recognize that eight dots in a ten frame corresponds to the number 8).</p>
<p>K.CC.C Compare numbers.</p> <ol style="list-style-type: none"> 6. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group for groups with up to 10 objects. 7. Compare two numbers between 1 and 10 presented as written numerals. 	<p>Every Image and Tool Talk involves a series of images where students consider what has changed between images. Are there more or less? How many more/less?</p>

Operations and Algebraic Thinking – K.OA

K.OA.A Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

1. Represent addition and subtraction of two whole numbers within 10. Use objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.
2. Solve addition and subtraction word problems, and add and subtract within 10, by using physical, visual and symbolic representations.
3. Decompose whole numbers from 1 to 10 into pairs in more than one way by using physical, visual or symbolic representations.
4. For any given whole number from 1 to 9, find the number that makes 10 when added to the number, by using physical, visual or symbolic representations.
5. Fluently add and subtract within five, including zero.

Students discuss the actions of addition and subtraction in every Image, Tool, and Number Talk, with teachers recording a variety of representations. Anchor Problems, Contextualized Practice Problems, and 3-Act Tasks invite students to create their own representations and learn from one another's representations through discussion. Journaling and reflection are opportunities for connections and metacognitive representations of the big ideas within addition and subtraction.

The Contextualized Practice Problems and Anchor Problems provide ample practice with word problems of every problem type. Numbers are offered within 5, 10, 20, and multidigit numbers.

The Buttons (4), Markers (8), and Peppers (10) Lesson Strings are explicitly focused on the "Combinations for Ten" strategy, with plenty of embedded practice in all routines. The two "Partners for Ten" games specifically target this standard. The Image, Tool, and Number Talks intentionally encourage composing and decomposing numbers.

Numbers and Operations in Base Ten – K.NBT

K.NBT.A Work with numbers 11-19 to gain foundations for place value.

1. Compose (put together) and decompose (take apart) numbers from 11 to 19 into ten ones and some further ones, and record each composition or decomposition by physical, visual or symbolic representations; understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

The Markers (5), Pizza (11), and Coins (15) Lesson Strings explicitly focus on the "Ten and Some More" numbers. Anchor Problems and Contextualized Practice Problems provide opportunities for students to work within 20, and also to work with multidigit numbers and notice connections between "10 and some more" and "20 and some more," and so on. The Teen Game provides additional practice.

Idaho Content Standards for Mathematics

– GRADE 1 –

Operations and Algebraic Thinking – 1.OA	
<p>1.OA.A Represent and solve problems involving addition and subtraction.</p> <p>1. Solve addition and subtraction within 20 involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, by using physical, visual and symbolic representations.</p>	<p>The Contextualized Practice Problems and Anchor Problems provide ample practice with word problems of every problem type. Numbers are offered within 5, 10, 20, and multidigit so students can solve problems of appropriate challenge. Students represent the operations with objects, drawings, and equations.</p>
<p>1.OA.B Understand and apply properties of operations and the relationship between addition and subtraction.</p> <p>3. Apply properties of operations as strategies to add. Examples: 1) If $8 + 3 = 11$ is known, then $3 + 8 = 11$ is also known. (Commutative property of addition.) 2) To add $2 + 6 + 4$, the second two numbers can be added to make a ten, so $2 + 6 + 4 = 2 + 10 = 12$. (Associative property of addition.)</p> <p>4. Restate a subtraction problem as a missing addend problem using the relationship between addition and subtraction.</p>	<p>All tasks in <i>Building Fact Fluency</i> are designed so students will discover, explore, and understand the properties of operations in addition and subtraction.</p> <p>The <i>Building Fact Fluency</i> toolkit was written to support development of fact fluency alongside and intertwined with conceptual understanding of addition and subtraction relationships. Every task aligns to this standard, and there are plenty of opportunities for first graders to use a variety of strategies to solve problems within 20 as they become increasingly fluent.</p>
<p>1.OA.C Add and subtract within 20.</p> <p>5. Relate counting to addition and subtraction.</p> <p>6. Demonstrate fluency for addition and subtraction within 10, using strategies to add and subtract within 20.</p>	<p>Every single task in the <i>Building Fact Fluency</i> toolkit aligns to this standard.</p> <p>The Image, Tool, and Number Talks build in a series that often involves counting up or down by some number and invites the connection to addition. The Shells, Bears, and Blocks Lesson Strings focus specifically on adding or subtracting 0, 1, or 2. Many of the games invite this relationship using dice, ten frames, or numerals.</p>
<p>1.OA.D Work with addition and subtraction equations.</p> <p>1. Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, <i>which of the following equations are true and which are false?</i> $6 = 6$, $7 = 8 - 1$, $5 + 2 = 2 + 5$, $4 + 1 = 5 + 2$.</p> <p>2. Determine the unknown whole number in an addition or subtraction equation relating three whole numbers, with the unknown in any position. For example, <i>determine the unknown number that makes the equation true in each of the equations</i> $8 + ? = 11$, $5 = ? - 3$, $6 + 6 = ?$.</p>	<p>The connections from Image to Tool to Number Talk invite much discussion about the symbolic representation of addition and subtraction. In addition, there are many opportunities to compare expressions, leading to equations such as $5 + 3 = 3 + 5$ and $7 + 3 = 8 + 2$, which invite relational thinking about the equal sign (as opposed to thinking the equal sign means “the answer comes next”).</p> <p>The Contextualized Practice Problems provide ample practice opportunities for every problem type, with unknowns in all positions.</p>

Number and Operations in Base Ten – 1.NBT

1.NBT.B Understand place value.

2. Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand:
- 10 can be thought of as a bundle of ten ones — called a “ten.”
 - The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.
 - The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).

The Combinations for Ten, Ten and Some More, and Make-10/Pretend-10 Lesson Strings emphasize the bundling of ones into tens, including numeric representation of tens and ones. The multidigit numbers in Contextualized Practice Problems encourage extension of these ideas into higher tens.

The Buttons, Markers, and Peppers Lesson Strings are specifically focused on decomposing and composing ten. Throughout the toolkit, use of ten frames, linking cubes, and Rekenreks encourage students to explore ten.

The Markers, Pizza, and Coins Lesson Strings contain problems, games, and tasks that encourage students to understand the structure of the teen numbers. In additional Lesson Strings, ten frames, linking cubes, and Rekenreks build on students’ understanding of ten and extend into the teen numbers.

Wherever appropriate, the Contextualized Practice Problems include a multidigit option that encourages students to extend their understanding of addition, subtraction, and place value into larger tens (e.g., number selection might include $4 + 6$, $14 + 6$, $24 + 6$).

1.NBT.C Use place value understanding and properties of operations to add and subtract.

4. Add whole numbers within 100 by using physical, visual and symbolic representations with an emphasis on place value, properties of operations, and/or the relationship between addition and subtraction;
- Add a two-digit number and a one-digit number
 - Add a two-digit number and a multiple of 10
 - Understand that when adding two-digit, combine like base-ten units such as tens and tens, one and ones, and sometimes it is necessary to compose a ten.
5. Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.

Wherever possible, Contextualized Practice Problems include multidigit number choices where students add and subtract within 100 in context. Students are encouraged to represent their thinking in a variety of ways and discuss the strategies they develop, as well as reflect on the bigger ideas through journaling and reflection.

The Plus and Minus 0, 1, & 2 Lesson Strings [Shells, Bears, and Blocks] provide several weeks of practicing one more and less in all routines, including the Making More and Less game. The Contextualized Practice Problems in these Lesson Strings invite students to extend their understanding beyond 20, and discussion among students solving a range of number choices, (e.g. $4 + 6$, $14 + 6$, $24 + 6$) would yield discovery of patterns around ten more and ten less.

Idaho Content Standards for Mathematics

– GRADE 2 –

Operations and Algebraic Thinking – 2.OA	
<p>2.OA.A Represent and solve problems involving addition and subtraction.</p> <p>1. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, by using physical, visual and symbolic representations.</p>	<p>The <i>Building Fact Fluency</i> problem-based lessons—Contextualized Practice Problems, Anchor Problems, and 3-Act Tasks—provide hundreds of opportunities for students to solve addition and subtraction word problems within 100 in all problem types and with unknowns in all positions. Students’ representations of their work are core to these lessons and to formative assessment within the toolkit.</p>
<p>2.OA.B Add and subtract within 20.</p> <p>2. Demonstrate fluency for addition and subtraction within 20 using mental strategies. By end of Grade 2, recall basic facts to add and subtract within 20 with automaticity.</p>	<p>Every single task in <i>Building Fact Fluency</i> is designed to build fluency within 20 while building conceptual understanding of addition and subtraction.</p>
Numbers and Operations in Base Ten – 2.NBT	
<p>2.NBT.B Use place value understanding and properties of operations to add and subtract.</p> <p>5. Fluently add and subtract whole numbers within 100 using understanding of place value and properties of operations.</p>	<p>The multidigit options of the Contextualized Practice Problems and Anchor Problems offer hundreds of opportunities to practice adding and subtracting within 100 using strategies based on place value, the properties, and relationships between the operations.</p> <p>The assessment strategies in <i>Building Fact Fluency</i>—including metacognitive journaling and reflection, student self-assessment, observations during games and problem-based lessons, and interviews—offer students multiple opportunities to articulate and explain the meaning of the operations, their properties, and solution strategies. In addition, all the Lesson String components (Image Talks, Tool Talks, Number Talks, Anchor Problems, Contextualized Practice Problems, 3-Act Tasks, and Games) invite discussion about the meaning of the operations and provide opportunities for students to explain their thinking in both writing and talk.</p>