

### WHY SHOULD WE REVISE OUR MATHEMATICAL THINKING?

<p><b>Become More Correct</b></p>	<p>This is the most prevalent reason why we might think we need to revise our mathematical work: to correct mistakes. But this is only one reason among many to revise. Many correct solutions would benefit from additional revisions. There is more to understand about mathematics beyond seeking a correct answer.</p>
<p><b>Seek New Insights</b></p>	<p>Our initial drafts could be viewed as our first way of solving a problem or our first way of seeing a mathematical relationship. Successive drafts could include alternative solution approaches, new ways of seeing, or alternative noticings.</p>
<p><b>Become More Precise</b></p>	<p>We can examine the ways that we talk or write about mathematics to see whether we can communicate what we mean in a more exact manner. There may be more than one way to interpret an early draft, and successive revisions can help with communicating more clearly.</p>
<p><b>Become More Detailed</b></p>	<p>In our initial drafts, we may assume that audiences know what we mean. Elaborating by adding additional details to fill in where we may have made assumptions about shared understandings can improve communication.</p>

TABLE 4.4

*(continued)*

<p><b>Become More Concise</b></p>	<p>Early drafts often have a degree of repetition and redundancies as we talk or write to work out what makes sense to us. Some sentences might be overly wordy. Revising allows us to winnow down to the essential points.</p>
<p><b>Become More Elegant</b></p>	<p>Mathematicians often express appreciation for a proof that has beauty and elegance. The aesthetic quality of an argument can be found in its simplicity, in its transparency, in the unexpected, or in visual appeal. Perhaps we can revise our thinking and represent it in a more elegant manner.</p>
<p><b>Become More Illuminating</b></p>	<p>We can review our work by trying to switch perspectives—from speaker or writer to audience member (listener or reader)—and consider how our ideas can be represented so that they are clearer to another person. We can add examples to help the ideas come to life for the reader.</p>
<p><b>Become More Convincing</b></p>	<p>If we are trying to justify why something is true, our argument may not yet convince a skeptic. When we develop our thinking, we first convince ourselves. A next revision could convince a friend. An additional revision could convince a skeptic, which includes making sure important ideas are supported (Mason, Burton, and Stacey 2011).</p>

TABLE 4.4

*(continued)*

<b>Become More Realistic</b>	If we are trying to write a story to represent a mathematical situation, we may first work on trying to make sure the mathematical relationships make sense. After reading the initial draft, we might realize that our situation is not realistic. A revision can address these contextual issues.
<b>Become More Illustrative</b>	An initial representation of our thinking might not have visual diagrams or the diagrams might not yet represent the mathematics and provide clarity to others.
<b>Become More Connected</b>	To see mathematics as a connected system of ideas, we could take another pass at our drafts to communicate how these ideas connect with ideas we have thought about in the past.

TABLE 4.4